INCREASING POWERS OF MATH EMPORIUM TUTORS

\[ T^0, T^1, T^2, \ldots, T^{(N-1)} \]

Tom Peters

Virginia Polytechnic Institute and State University
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PREFACE

A Note from the Author
I began my mathematics teaching career at Blacksburg High School in 1971. There were no hand held calculators, much less computers, available to our faculty and students. Instead, students in my Advanced Algebra and Trigonometry courses were taught how to do complex calculations on a slide rule. Before you write me off with the dinosaurs, let me assure you that there were no "slide rule-dependent" students in those days.

In the spring of 1998, three years after completing my graduate work, I was offered the job of supervising the Math Emporium at Virginia Tech. There was very little by way of job description for this new position, but I was awed at the sight of 500 computers in one room. The journey from slide rules to computers has been a wonderful, but also challenging, adventure. I have grown and evolved with the Emporium into a new dimension in the teaching of mathematics.

While seeking and collecting literature and on-line materials for orientation and training of our tutoring staff, I have gradually come to realize that the traditional roles of mathematics educator and math tutor are changing in ways stimulated by new technologies. Mathematics educators, who are juggling active research programs and curricula,
are struggling in a quest to stay abreast of the rapid development and transition within the field of the teaching of mathematics. It is this quest that led and inspired me to write this booklet.

The Effects of New Technologies
The dynamics of teaching mathematics have undergone gigantic changes with the implementation and progression of the computer and multitudinous software programs in the last two decades. These changes can be observed in all strata of the educational spectrum, from pre-school to graduate level mathematics.

The effects of using advanced technology in mathematics education demands evaluation. Sometimes the evaluation process reveals results that "sting" like a bee. Recent research has confirmed that placing a calculator or the keyboard and mouse of a powerful computer into a student’s hands, without proper instruction and motivation, does not necessarily guarantee an improvement in the quality of education. With the use of advancing technology, we have simultaneously created (to some degree) a calculator-dependent society. For example, many students are unable to graph a quadratic equation without the use of a graphing calculator. This is not to say that these tools are bad. Quite the contrary, given the proper pedagogical and instructional setting, these tools can elevate us to a much more advanced and sophisticated level of instruction.

In the industrial world, automation has created a demand for more highly skilled professionals with computer-related skills in the job market. Such is also true in the teaching of mathematics. Costly computer labs equipped with the latest
Most powerful computers are no better than the professors, instructors, and tutoring staff that command them. Thus, we are now confronted not only with the task of producing teachers and tutors that master the subject matter but we must also prepare our teachers and tutors with the computing skills needed for teaching the subject.

The Need for Help
Teachers and students are often in need of an instant resource for help while utilizing new technologies and computer assisted instruction. I painfully remember the difficulties encountered in separating the syntax of Matlab from that of Mathematica. A simple misplaced period or parenthesis can determine the difference between success and failure. No doubt, at some time in your life, you have experienced some degree of frustration in trying to run a particular program or implement some command. Have you ever tried getting help from the friendly program menu that turned out to be your foe rather than your friend? Or, better yet, have you ever consulted the on-line help? Perhaps some fellow teacher or co-worker lent you a tutoring hand to guide you through your despair.

Such needs have dictated the evolution of the teaching of mathematics to include a tutoring lab environment with "on the spot" staff assistants. The titles of these assistants along with their tasks may vary. Some common names are lab assistant, TA (teaching assistant), and GTA (graduate teaching assistant) but they are all essentially tutors. It is not only the source of hardware or software that constitutes success in this environment, but also the immediate feedback of friendly, knowledgeable tutoring staff with
hands-on assistance that leads to the cutting edge of successful transfer of mathematics concepts.

Virginia Tech’s Math Emporium

In hot pursuit of better ways to integrate math education and technology, the Mathematics Department of Virginia Polytechnic Institute and State University has given birth to the Math Emporium. It is the brainchild of Dr. Robert (Bob) Olin past chair of the Virginia Tech Math Department, Dr. Mike Williams (who was Associate Vice President for Information Systems and Research Computing at the time of its origin) and other members during Olin’s term of office. In its short life span (seven years, as of this writing) it towers as a leader in the structure of teaching and tutoring undergraduate mathematics and demands precedence and recognition within its field.

University-wide budget constraints have given rise to an increase of on-line classes, larger than normal sized classes, and consolidation or elimination of undergraduate core courses in the curriculum. The Math Emporium has enabled the Math Department to face tighter budget cuts while maintaining educational excellence.

The Math Emporium is an annexed building that houses over five hundred Macintosh computers loaded with the most up-to-date software. It is serviced by two talented and proficient computer programmers. The Emporium serves as the base for math tutoring services for all mathematical courses in the core curriculum. In addition, the Emporium is the site for lab-based programs, lab-orientation lectures,
and assignments that supplement engineering calculus, business calculus and life science calculus courses.

The Math Emporium operates seven days a week, twenty four hours a day. By way of disclaimer, the Emporium is not a closed set for mathematicians. It opens its doors for other disciplines to take advantage of its facilities. For example, an entomology course administers exams and finals at the Emporium. Many non-math majors take advantage of the resource to do group study and work on various projects at its pods.

The Math Emporium offers an abundance of memory laden computers, powerful operating systems, and the latest software packages. Yet, its strength is embedded in the power of its unique, diverse, and competent tutoring staff. The tutoring staff consists of professors, instructors, graduate students, and undergraduates. It is truly a melting pot for the Math Department and its students.

The original stimulus for writing this booklet was to empower the Math Emporium staff at Virginia Tech to become most effective. We believe, though, that this booklet presents ideas, procedures, and suggestions that would be useful to many similar-minded groups. It should serve to enhance the experience and effectiveness of teachers and tutors.

Acknowledgements
The writing of this training booklet for Math Emporium tutoring staff was as complex as the operation of the Math Emporium itself. There were many challenges to face beginning with the initial design of the basic plans to the
implementation of the project. Its final product is the mathematical union of many tutors and their labors, none of whose contributions should be unnoticed.

Although I rightfully claim to be its main author, I wish to acknowledge the following staff which contributed very much to its design. First, I wish to thank Linda Powers who was indeed the first to concur with my ideology for the need of such a booklet. Secondly, I wish to thank my Emporium Manager, Chuck Hodges, who encouraged me, gave me constant support and reviewed the booklet during this endeavor. I also wish to thank Dr. John Rossi (current head of the Math Department at Virginia Tech) who supplied the financial support for this writing. Without the expertise and untiring efforts of Dr. Mike Williams, the Math Emporium would never have received such prominence. I wish to extend much thanks to the excellent work of Terri Bourdon, Debbie Smith, Dr. James Washenberger and a host of other excellent math instructors with whom I have been privileged to work and share ideas while in the Math Emporium. A special thanks is due my sister Dr. Wanda Dann from Ithaca College, Ithaca, New York, whose skills and suggestions rendered as my main reviewer, I found invaluable. My express thanks to Louise, my precious wife, whose confidence and patience pushes me to new limits. Last but not least, I wish to thank all of those wonderful student tutors and tutees that have so graciously enriched my career.
Within the field of mathematics, many different types of tutoring models exist in a diverse set of operative programs. These are contingent on the particular needs and budget limits of the individual or the institution. Some models are conventional and some unconventional. In this introductory chapter, the various models are classified according to function and purpose. The learning theories, on which these models are based, are also presented.

**Personal One-to-One**

This is the conventional model of tutoring that has been used for many, many years. Some parents, guardians, or even students, hire their own personal tutor. This establishes a one-to-one relationship between the tutor and tutee and sets up a long term base for tutoring. Its weakness is its extra demand on the student's already-taxed financial base as well as using up the student’s homework time. The tutor is paid by the student (or other outside source) and therefore responsible to the same.

**Institutional One-to-One**

This model is an adaptation of the Personal model above. In recent years, college and university programs have
instituted procedures to support students with a personal tutor. The tutee may have to apply for such help and, upon proper qualification and procedure, then be assigned a tutor. The tutee will often report to a designated area for the tutoring session on a long term basis. The college or university covers the fee for such services and the tutor is responsible to the institution.

Instructor Based
This model is most effective for students who need very limited help with a particular assignment or lab. It is incorporated within the office hours of the professor or instructor and made available by appointment or "drop in." Instructor based tutoring is often the very best source for assistance and information relative to a specific course. It is insufficient, however, for those students who need more time as they struggle with learning the course material. This situation is worsened if the class size is large or if students have schedule conflicts with the instructor's office hours. Clearly, the funding for the instructor based model is through the instructor’s regular pay.

Special Group Based
Some universities offer additional academic support for groups who qualify. At-risk students may be offered special tutoring sessions. Athletes whose team practice and travel schedules can interfere with success in the classroom may be provided with study sessions facilitated by special tutors. They are assigned an instructor or group of instructors who personalize the instruction and serve as unique tutors for the entire group. The weakness of this model is that it serves only a small percentage of the
student population and is funded at the extra expense of the college or university.

*Graduate Assistant Based*
Some math courses require their students, after having attended the traditional lecture, to attend a class where graduate students work with and assist the students with their class assignments. An example at Virginia Tech is the Vector Geometry recitation class. In the recitation class, the students can obtain one-on-one help from graduate students on how to solve various problems that challenge them. The strength of this model lies in the base of knowledge and expertise of gifted mathematics students who have mastered the course content. This approach is course specific and although attendance is mandatory, some students still tend to give up on attending recitation sessions. This model is funded by graduate student stipends.

*Lab Based*
This model provides an excellent source of graduate and undergraduate help in an unconventional smorgasbord environment. Students can walk in, sign up, and receive one-on-one or sometimes group assistance depending on the tutor-tutee ratio. This is an excellent source of help for students enrolled in undergraduate math courses. The weakness is that it can serve only a small percentage of the student population and some students feel intimidated (or are too embarrassed) to attend such help sessions. Attendance is not mandatory. This model is funded by the university through stipends and wage salaries.
Technology Based
A wide assortment of video classes, video-taped lectures and computer generated lectures are now available in most mathematics curricula. Such media provide a tremendous supplemental resource for most undergraduate courses. The obvious weakness is that it provides no basis for personal interaction with a tutor to ask questions and obtain answers. It would be naïve to suggest that such lectures leave no further questions in the student’s mind.

Emporium Based
This is the newest and most unconventional model in use in the field of mathematics education today. The Emporium model is an integral coordination of computer assisted instruction modules with spontaneous, on demand, one-to-one tutoring in a computer lab setting. Students work with computer assisted instruction modules and complete assignments, quizzes, and exams in the Emporium. Tutors are constantly available to help them with questions regarding use of the computer and software and to provide assistance with content and problem solving concepts. Its strength lies in the tremendous source of instantaneous help relevant to most of the lower level mathematics courses.

Virginia Tech's Tutoring Models
Virginia Tech has a dichotomy of tutoring models. The first is the Lab Based model. A staff of qualified and instructor-recommended graduate and undergraduate mathematicians is available for individual or group tutoring during certain hours on a weekly basis. (At this writing, lab based tutoring is available Sunday through Thursday nights from 6:00 - 9:00 p.m.)
Students enrolled in first-year math courses are free to drop in at will and take advantage of tutoring offered in this program.

The tutors can often select which students to help, based upon their familiarity with the subject matter. Lab based tutoring demands a broader base of course knowledge than other conventional models but still retains some freedom. For example, one tutor may feel comfortable with integral calculus but not with differential equations. Since duties can be distributed in this fashion, tutors are not necessarily required to "cover all the bases."

In this particular model, a tutor may or may not encounter the same tutee in the course of a semester. It is therefore extremely necessary to establish as much of an interactive relationship as possible with the tutee as time permits in this situation.

Virginia Tech's second tutorial model is the Emporium Model. Students are required to work in the Math Emporium on a broad base of subjects that cover all first-year mathematics courses offered in the curriculum. To name a few, they are enrolled in Math 1015 Pre-calculus, Math 1016 Differential Calculus, Math 1114 Linear Algebra, Math 1224 Vector Geometry, Math 1205 Differential Calculus for engineers, Math 1206 Integral Calculus for engineers, Math 1525 first semester Business Calculus, Math 1526 second semester Business Calculus, Math 2015 Life Science Calculus, Math 2214 Differential Equations, and Math 2224 Multivariable Calculus.
Some of these courses are offered strictly on-line and students are required to take graded on-line quizzes and exams in the Math Emporium. Well-trained proctors administer student exams and finals for various mathematics courses. In the Fall-2003 semester, over 500,000 quizzes and exams were taken and linked to appropriate databases.

The Emporium tutor must be prepared for spontaneous interaction with students on a wide variety of computing and mathematics topics and skills. Thus, demands on the tutor in the Emporium model are much higher than in either the Class Based or Lab Based models.

*Learning Theories*

Tutoring models are based on principles established in learning theories. What is a learning theory? A learning theory is a well-substantiated explanation of how people learn. At the time of this writing, there are more than a dozen learning theories.

Instruction (and tutoring) techniques are often associated with a specific learning theory. Two learning theories are of particular significance to Math Emporium tutors. First, the Brain-Based theory contends that all learning takes place inside the brain. In this model, the brain is considered as functioning like a powerful, memory-laden processor that can assimilate, store, process and retrieve data. Since everyone is born with this device, everyone can learn.

*On-purpose Associates* (an on-line resource for educators) lists instructional techniques associated with brain-based learning:
1. **Orchestrated immersion**
   Creating learning environments that fully immerse students in an educational experience.

2. **Relaxed alertness**
   Trying to eliminate fear in learners, while maintaining a highly challenging environment.

3. **Active processing**
   Allowing the learner to consolidate and internalize information by actively processing it.

This theory lends itself to the infrastructure of an Emporium that orchestrates learning through the environment of computerized instruction. The atmosphere is relaxed through the assistance of highly trained and friendly tutors who lend "on-the-spot" help when needed. Students are encouraged to actively process and internalize information by working through lesson modules, labs, practice and graded quizzes.

The second learning theory of relevance to our discussion is the Neuroscience model. *On-purpose Associates* describes the Neuroscience model of learning as a nebulous web system with the brain interconnected to the circuits of the central nervous system:

*The brain is not a computer. The structure of the brain's neuron connections is loose, flexible, "webbed," overlapping, and redundant. It's impossible for such a system to function like a linear or parallel-processing computer. Instead, the brain is better described as a self-organizing system.*

Each time a concept or behavior is learned it expands the nerve endings to branch out for more such experiences.
Cultural experiences and language communication provide the stimuli and catalysts for educational growth and perception in this model. Math Emporium tutors should be aware of Neuroscience learning theory because of its significance for peer-based teaching and interaction between tutors and students. The human body's marvelous brain and central nervous system house a web of mathematics understanding. The language and visual communication between tutor and tutee can provide a catalyst for expanding the student's math web. Through the use of computerized modules and lesson quizzes, the web is expanded even further.

Topics for Discussion

● Name two types of tutoring models used at Virginia Tech's Math Emporium.

● What is one major variable between these models?

● In your own words, contrast the differences between the Lab Based Model and the Emporium Based Model.

● What learning theories are related to the Emporium Based model? Explain.
What Is an Emporium Tutor?

The Math Emporium staff consists of a variety of tutors including professors, instructors, graduate and undergraduate students. The Math Emporium tutor is required to have a vast knowledge of all the mathematics courses in the core curriculum and their computer applications. In essence, this means the Math Emporium tutor must be a jack of all math trades. As preparation for this task, all members of the staff are required to attend a training workshop.

A Need for Knowledge Breadth

A question you might ask is, "Why is it necessary that a Math Emporium tutor have a vast knowledge of all math courses in the core curriculum?" The answer lies in the fact that many first-year mathematics courses offered in the curriculum require students to work with on-line materials in the Math Emporium. Students may encounter difficulties in the presentation of on-line materials or while solving problems in their practice quizzes and are encouraged to work multiple quizzes until they master the course content. As a result, they generate thousands of such quizzes each week. It is therefore crucial to their learning, that they receive proper assistance relative to the problems in their respective courses.
Other students are assigned labs that supplement the course content which require computer assistance. Many of these labs are preceded by an orientation lecture given by a qualified instructor. These are administered in the Math Emporium to encourage students to work on the assignment at the Emporium computers immediately after the lecture. Such is the case with the engineering calculus courses. These students often encounter difficulties with transferring data properly into the computer program, executing commands, and setting up proper solutions.

Business calculus students work through on-line learning modules and are required to take their quizzes at the Math Emporium. These modules contain sample problems generated on the computer and therefore demand their mastery. Many of these problems must be solved using Microsoft Excel®. Here again students encounter difficulties in transferring data and setting up proper solutions. The Emporium tutor must be capable of guiding the students through these problems with the assistance of the software.

Vector Geometry students are required to take on-line quizzes that correspond to their lessons. They may generate thousands of questions in a given week while practicing their quizzes on-line. The Emporium tutor must have full command of the subject matter to successfully guide the tutee through the problems.

In addition to all of the above, many students come to the Math Emporium to work on their homework assignments in integral calculus, differential equations, and multivariable
calculus. The Math Emporium tutor must have a working knowledge of these subjects as well.

The Math Emporium tutor must often double as a test proctor. There is a designated testing area in the Math Emporium where students take their exams upon completion of their lesson modules. The efficient tutor must initiate exams, stay active, alert, and circulate with a watchful eye to prevent cheating.

**Consistent Preparation**
A Math Emporium tutor has been pre-qualified by mastering the subjects. For graduate and undergraduate students, this is accomplished in earlier course work. Nonetheless, a little "brushing up" on the modular topics often determines the difference between a good Emporium tutor and a poor one. No matter how proficient you may be in your mathematics, no one can breeze through the abundance of these topics without consistent review.

No instructor would dare to walk into the classroom after having given a homework assignment and not properly prepare for possible questions. In anticipation of such questions, the instructor or professor will work through each problem either mentally or manually -- thus being well prepared before meeting with the class. Being a successful Math Emporium tutor requires no less. It is imperative that you familiarize yourself with questions that will inevitably arise. Confidence and rapport with students rests upon this principle.
At Virginia Tech, the course coordinators are responsible and required to provide sample problems and solutions for the quiz materials covered in their respective courses. These are collected and compiled into a manual and serve as a valuable resource guide for tutors "on the floor" in the Emporium. These should be kept privately in a designated area and not allowed to fall into the hands of students. The Math Emporium tutor should routinely work through this invaluable guide as one way to become familiar with course content and sequencing. In fact, I strongly suggest the tutor begin each new week with study of the manual.

In addition to the Emporium Lesson Manual, on-line lesson modules and quizzes are available for tutors. In down times, when it is not busy on the floor, the tutor should take advantage of the opportunity to work through the lesson modules and practice quizzes. They may not be as much fun as computer games but they will prepare you as a better tutor. Remember, you will be evaluated on your performance as a tutor by your Emporium supervisor. Supervisors tend to frown upon those who consume their time in non-job related activities.

*Guiding a Student to Understand a Concept*
The Math Emporium tutor must be able to render help with any core math subject at any particular moment of time. The signal for help in the Emporium is a red cup. Student pods are equipped with red cups that can be placed on top of the computer monitors. When a student puts up the red cup, this is a call for help. Upon answering the cup, it is most important that you politely request the student to take it down. This simple gesture avoids calling some other tutor to the pod and conserves valuable time.
Time is the crucial variable. In most conventional tutoring models, the tutor is given a much longer segment of time to get to know the tutee, become familiar with course materials, diagnose tutee weaknesses, and build upon their strengths to close the learning gaps. The Math Emporium tutor, on the other hand, must do all of this "on the spot" and in a spontaneous manner -- sort of *instantaneously*.

In other words, the Math Emporium tutor must support an instantaneous rate of change in the learning process. How can this be done? While there is certainly no given formula that one can offer to achieve such a magnificent goal, there is a mathematically sound way to reach this objective: examine and use the data on both sides of the point of contact to achieve the goal. This is analogous to the approximation of the instantaneous rate of change using an average rate of change, as illustrated in Figure 1.

Prior to the moment of contact, the tutor must examine and become familiar with the sequence of topics in the course and the types of problems the tutee will encounter in the lesson modules and on the quizzes. This will enable the tutor to draw upon the content and sequential structure of the course to guide the tutee through the problems and find proper solutions. This also serves to prohibit the tutor from introducing concepts above the current level of course instruction. For example, the student may be required to find the derivative of a function by the use of limits. The student may not yet have been introduced to the power rule and to use this concept prior to its introduction in the course would only defeat the purpose of the lesson module and serve to confuse the tutee.
Immediately after the moment of contact, the tutor must carefully analyze the student’s work and attempt to diagnose misconceptions or lack of knowledge in course content. By becoming very familiar with the lesson modules, the tutor may be able to refer the tutee back to the proper lesson module and examine it to extrapolate needed data. This is an invaluable way to arrive at proper mathematical formulas such as rules for differentiation, integration etc.

**Figure 1. Using data on both sides of point of contact**

<table>
<thead>
<tr>
<th>Before point of contact</th>
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</thead>
<tbody>
<tr>
<td>What is the sequence of topics in the course?</td>
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<tr>
<td>What problems are students likely to encounter?</td>
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</table>

<table>
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<tr>
<th>Point of contact</th>
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<tbody>
<tr>
<td>Red cup on top of monitor signals student need for help.</td>
</tr>
<tr>
<td>Student asks question.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>After point of contact</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diagnose the basis of the confusion.</td>
</tr>
<tr>
<td>Guide the student to an understanding of the concept.</td>
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</table>

By using the data on both sides of the point of contact, and through much practice, the tutor will become very skilled at bridging the learning gap and guiding the tutee to satisfactory solutions.
A basic premise to remember is **Guide -- but do not give.** An old adage advises that, "If you give a man fish you have fed him for a day, but if you teach him how to fish you have fed him for a lifetime." *The purpose of the tutor is not to show the student how much you know but rather to build confidence in how much he or she knows.*

**The Importance of Interest and Enthusiasm**

Qualified tutors are not only knowledgeable of mathematics but also they must enjoy working math with other students. *Tutors are indeed a reflection of the face of the math department.* It has been said that first impressions are lasting ones. It is therefore crucial that you be cordial and friendly. As an Emporium tutor, you are not afforded the pleasure of establishing a long term relationship with the tutee. Therefore, you must be quick to display your care and concern for the students. The Wal-Mart slogan may be a good one, “How May I Help You?” -- asked with a friendly smile.

In the book entitled *Tutoring Matters* by Rabow, Chin and Fahimian there is much emphasis on building relationships between tutor and tutee. The instantaneous tutor must strive to build this relationship on the spot. Perhaps the most valuable contribution of the Math Emporium tutor is a display of interest and enthusiasm.

Rabow says, “It is crucial that you show your passion for tutoring by showing interest in your student’s work and in their lives.....Tutees know that you cannot start loving them right away, (human relationships do not work that way), but your obvious love for your work and interest in them will illustrate your potential for bonding with them.”
Drop Your Baggage at the Door
The Math Emporium tutoring staff is made up of individuals who often work under high levels of stress. Some are instructors bogged down at times with labs or exams to grade. Some are graduate students with class projects in preparation for dissertations. Others are undergraduate students with heavy class loads, assignments, and exams. It is not uncommon to report to work when you are having a bad day. Always remember please, you must leave your cares and worries outside the doors of the Math Emporium.

While addressing baggage, there are lockers located at the entrance of the Math Emporium. It is most important that you lock up your valuables and book bags upon arriving at the Emporium so as to preserve precious working space at the command module and student pods.

When you walk through those Emporium doors, you must be transformed into a loving, caring, math wizard whose only concern is to guide students into a successful experience in solving math problems. Your papers and assignments can wait but your assistance cannot. Remember, while at the Math Emporium, you will be expected to render your very best.

Undergraduates
If you are not capable of keeping up with your own course materials to the point that you can concentrate on helping students while at the Emporium, perhaps this job is not for you. If you are working so many hours at the math Emporium that it is preventing you from keeping up with
your class assignments, then perhaps you should back down on your Emporium hours.

Poor staff members consume themselves in their own activities, grading papers or doing homework problems. This is selfish, to say the least, and not exemplary of a good Math Emporium tutor. Engaging in your own activities sends a subliminal message to the tutee that you do not wish to be bothered with them. This same pitfall occurs when you congregate in a circle with fellow workers and exclude the tutee. They become exterior to the circle. The good tutor circulates well and stays alert to cups and student needs. One of the questions on the Math Emporium tutor evaluation sheet is, “Does the tutor make herself readily available to student questions, circulate well, and spend time on Math Emporium related activities?”

**Personal Responsibilities**

Wear your lanyard at all times. This is the way tutees can identify you as being a professional Math Emporium tutor. If you have not been issued one, ask your supervisor who will be happy to supply you with one. Once issued, it is your responsibility to keep the lanyard with you at all times and display it when you report for Emporium duties.

Never miss a shift without planning for a substitute. One mark of a professional Emporium tutor is to know when he or she will be absent and plan ahead to have his or her shift covered. You should first notify the Emporium Manager and seek permission to be absent. (At the time of this writing, the Emporium Manager at Virginia Tech is Chuck Hodges.) The manager provides a list-serve, through which any Emporium tutor can advertise and search for substitutes.
well in advance of the schedule. There are busy days when being short-handed on staff makes life very difficult for everyone involved in the Emporium.

Some tutors may have health problems which cause them repeated difficulty and multiple absences. Others may have a number of job interviews in a particular semester requiring a number of absences. In these cases, the tutor should arrange for a ‘standing alternate’ who would be willing to substitute for them on a regular basis and when such occasions arise.

There are emergencies that can catch all of us off guard but please be professional enough to send an e-mail or call the supervisor for proper notification. I would caution that too many emergency substitutions provide reason for replacement. Remember, it is your responsibility to find a substitute and not that of the Emporium staff.

Summary
The Math Emporium tutor is a rare creature who possesses all of the math skills necessary to help in a variety of subjects. He or she is an instantaneous tutor who can take advantage of available data on both sides of the point of contact with a tutee and is able to establish an instantaneous rate of change in learning behavior. The Math Emporium tutor is a friendly and caring individual whose major concern is to assist students in the mastery of mathematics concepts. Tutors are expected to take personal responsibility by wearing a lanyard and planning ahead for substitutes.
Topics for Discussion

• What is one major factor that separates a good Emporium tutor from a poor one?

• Name three ways to improve your status as an Emporium tutor.

• Why is it important to be friendly and cordial to all tutees?

• What is wrong with being in a circle?

• Define the term "instantaneous tutor" using the concept of the derivative.

• Who is responsible for seeking a substitute and how is this done?
3

T² Tutoring Techniques

One cannot overemphasize the importance of knowing the subject matter well and being able to execute smoothly while on the floor assisting students. The interchange between tutor and tutee is a key factor in the process. The efficient tutor must capitalize upon the information provided by the tutee at the point of contact. This chapter presents a number of tutoring techniques that can help you be successful in this endeavor.

Tutoring techniques are related to and associated with instructional theories. An instructional theory is a set of strategies implemented for the purpose of supporting good student attitudes and learning. For example, Gagne's theory of instruction demands that all cognitive reasoning be centrally processed. Gagne's set of strategies can best be summarized as:

- State objectives clearly.
- Use embedded questions.
- Use examples and non-examples.
- Provide practice and feedback.

The tutoring techniques presented in this manual adhere to Gagne's strategies.
Concept Identification
Most often there is a learning gap between the student’s comprehension and the concept at hand. You must build a bridge between the two. This requires identifying exactly what the key points of information are that the student lacks. Without this information, the student is unable to proceed with the problem. Let me assure you that this is the foundation upon which to build any meaningful transfer of learning in the entire session. Too often the tutor will skip this step and proceed to offer a solution for the problem. In so doing, the real issue of constructing a learning bridge has been evaded.

Written Clues
One way to identify a student’s learning and information gaps is by careful examination of the student’s work written on paper or copied on the computer screen. For example, let us say the student has been asked to identify the number of solutions of a quadratic equation. In the following example, look for clues as you examine this tutee’s work.

How many real solutions does the equation
-4x² + 2x -7 = 0 have?

John has written:

\[ x = \frac{-2 \pm \sqrt{2^2 + 4(-4)(-7)}}{2(-4)} \]

\[ x = \frac{-2 \pm \sqrt{116}}{-8} \]

X has 2 real solutions.
It is obvious that due to a wrong sign under the radical, John needs a review of the quadratic formula. It appears that even though his answer is incorrect his concept is right. His gap lies in the memorization of the formula. The tutor should simply refer him to the correct form of the quadratic formula in the lesson module and suggest that he rework the problem based upon the correct version.

Let's look at a different scenario. Susie is working the same problem. Upon asking to see her work, the tutor sees that she has none. The above technique is now certainly not applicable. This situation leads us to the next technique for seeking conceptual errors.

**Intelligence-Seeking Conversation**

The insecure student is sometimes hesitant to admit that he or she does not understand some concept or has overlooked some important step. Like a smart detective, you must discreetly look for clues -- prying out information without being offensive. What is stored in the mind will inevitably be revealed by the speech. By engaging the tutee in intelligence-seeking conversation the tutor will cleverly discover clues concerning learning gaps.

The Tutoring and Academic Success Center (TASC) of Three Rivers Community College in Norwich, Connecticut, offers the following hints for engaging students in intelligence-seeking conversation.

- *Ask open-ended questions.* By posing questions that require more than a yes/no response, you encourage the tutee to start thinking. Some examples are:
Where do you think we should start on this problem?  
What exactly are we trying to find as an answer?  
Do you know any formulas that might help us find a solution?  
What are the steps involved in working this problem?

- *Ask probing questions which induce the student to contribute.* Some examples are:  
  What would we have if we took the derivative of this function?  
  What does the second derivative of this function yield with respect to its graph?  
  How does this output correspond to what is given on the graph?  

- *Rephrase questions when needed.*  
  Repeat your question in a slightly different manner.  
  Reword your original question.  
  Break your original question into smaller parts.  
  Change your voice inflection to emphasize meaningful points.

One way to encourage students to participate in intelligence-seeking conversation is to *ask leading questions with obvious answers.* One of my colleagues, a well-qualified math instructor and course coordinator for on-line classes offers the following scenario, based on the quiz problem shown in Figure 2.

(NOTE: This is a worst-case scenario. Most students will not be quite so clueless. The point to remember is that the more students say, the more they will remember. Even though we all like to think that if we say it, they will hear it, it rarely works that way.)
You: What did you try?

Student: Nothing.

You: Read the question to me.

Student: The surface area, S, of . . .

You: No, not the problem, the QUESTION.

Student: Which of the following is the best estimate for the RATE OF GROWTH . . .

You: Stop there. Do you know a generic way
to say "rate of growth?"

Student: Pause... Confused.

You: Rate of ch . . .

Student: Rate of change.

You: Is this an instantaneous or an average rate of change?

Student: I don't know.

You: Take a guess - there are only two choices. Which one would apply when r EQUALS 196?

Student: Instantaneous rate of change.

You: Is the function represented by a formula, a table of values, or a graph?

Student: A formula.

You: Do you remember how to find an instantaneous rate of change for a function when it is represented by a formula?

Student: Is that when you make a table around the number and subtract?

You: Yes. You are finding the average rates of change on either side of 196. Then What do you do with these numbers?

Student: Add them and divide by 2.

You: Nice job; now you have it. Go ahead and finish the problem on your own.
On-line Teacher Tips
Several undergraduate math courses are offered as on-line classes. Naturally, the major source of assistance and communication between the course coordinator and the student is to be found on-line. Contained within the instructor’s weekly syllabus are important announcements and valuable lesson tips with respect to problem solving strategies as related to the respective modules. An example is illustrated in Figure 3. It is most important that the tutor become familiar with the lesson tips and refer students to them when necessary.

Math 1016 - Tips for Week 6

The first lesson in 5.1 shows you how to find the absolute extrema for a function represented by a formula. This process is called “optimization”. Remember that the absolute and local extrema can be different for a function. For example, a local maximum point on a graph only needs to be higher than the other points around it. An absolute maximum point must be higher than (or as high as) all of the points on the graph. Fortunately, finding the absolute and local extrema have a lot in common. You still need to find where the first derivative is zero or undefined. Then, for absolute extrema you simply add any endpoints to your list of possibilities.

In the second lesson, you will develop your own functions to describe practical situations. Then, simply repeat the techniques from the first lesson to “optimize” your function. Of course this can be more challenging, especially if you have often found applied problems to be difficult to solve. Work through every example in the lesson. Even though each application is different, you will find that the more problems you work, the more confidence you’ll have when you see new ones in the quiz.

Data Note: While most of the quiz problems will be new applications, you may see one that is a variation on Example 2 in Module 5.1, Lesson 2. In the example, the

Figure 3 Lesson Tips
Notes — Clues and Hints

An innovative instructional tool, called Notes, is now available to help students seek a hint or a special clue about a particular kind of problem or problem-solving technique. The Notes tool, as seen in Figure 4, has been developed through the cooperative efforts of math academia and computer programmers. This tool enables links from certain problems to a hint or note. At the click of a mouse, the student has access to an additional resource for insight and problem-solving strategies. The tutor should encourage students to optimize their use of this valuable learning tool.

Figure 4. Note example
The note or clue shown in Figure 4 would be made available to a student after the test had been submitted for grading. By reviewing the problem and the hint, the student is able to gain an additional problem solving strategy with which to solve similar problems that he or she may encounter in the future. In this particular example, the student is shown that this kind of problem can be solved by applying the product rule.

*Listening Links*

The purpose of intelligence-seeking conversation is to locate learning gaps. It is the tutee who has the knowledge gaps and by learning to be a good listener you will discover them. The student will often reveal the source of his or her confusion by answering your questions. You must sometimes remain silent and give the student a chance to respond. Then, link the student's response to concepts underlying the problem at hand.

Listening requires much patience and practice and is difficult to master. At busy times on the floor when many red cups are up and other students are waiting for help you might be tempted to rush this process. Please keep in mind that simply giving answers is not good tutoring. Taking the time to listen is time well spent in the learning cycle.

What about when it’s really busy? One instructor offers the following advice:
I know you’re all thinking, “Yeah, right. This is fine when it’s slow, but what about when the place is full and red cups are everywhere?”

Of course, the main reason for this is that students have waited until the last minute and are expecting quick, easy answers. Avoid the temptation to write down a solution for them so that you can move on to the next ten red cups.

Actually, writing a solution for the student does the opposite of what you would expect. What usually happens is that they call another helper over to explain what you wrote. Instead of saving time, it really takes longer.

I experimented with timing one afternoon a few weeks ago. While imagining that it was very busy, I tried to be aware of the amount of time it took for me to guide the students toward their own solutions. I realized that it took the same amount of time to ask them questions, and then leave them with some obvious hints, as it would have to solve the entire problem for them.

As another example, let us now return to Susie’s scenario. Recall that Susie has been asked to identify the number of solutions of a quadratic equation but she has nothing written down for us to examine.

Applying our techniques for intelligence-seeking conversation, the tutor asks, “What do you know about the quadratic equation and finding roots?”

The tutor waits for Susie to answer. She responds by spitting out the formula.

The tutor then discreetly asks, “What does the sign of the number under the radical sign in the
quadratic formula tell me about the number of solutions?"

Once again, the tutor pauses. After some hesitation, Susie admits, "I don’t know."

By asking leading questions and listening to the answers, the tutor now knows that Susie does not understand the concept of the discriminate. At this point, the tutor can refer her to the proper lesson module and unit number for learning this concept.

Extended Explanation
An extended explanation may be necessary for the tutor to help the student bridge a learning gap. After identifying the confusing concept, it may be necessary for the tutor to give a mini lesson on the topic. For example, the student might need assistance in finding the cross product of two vectors by finding the determinant of a three by three matrix. This is a topic that will require an extended explanation. If when applying the intelligence-seeking conversation mode you encounter a great deal of frustration on the part of the student then switching to this mode might be more advisable. If the Emporium is busy, the tutee really needs lengthy help and the tutoring lab is open, the Emporium tutor should encourage the tutee to seek help in the tutoring lab.

Charts, Diagrams and Graphs
One of the most dramatic advantages of computer assisted instruction is the enhancement of visual learning. For example, the slopes of secant lines and tangent lines, which are basic to introducing derivatives, can readily be accessed
on the computer screen. With the use of a straightedge and color coded graphs of computer-generated functions, the Emporium tutor can wave a magic wand and perform miracles before the very eyes of tutees. Computer graphics lends a wonder world of magic and imagination to mathematics.

There are instances, of course, when the old fashioned drawing of a diagram is a quick and easy technique for illustrating a concept. For example, hand-drawn sketches to show situations involving distance, rate and time problems are more than worthwhile. Likewise, hand-drawn sketches that illustrate resolving two forces with corresponding magnitude and direction are a proven resource for increasing student understanding.

*Conceptual Convergence*

Many of the tutoring techniques described above are directly related to an instructional theory of my own thought and observation, called Conceptual Convergence.

When I was a younger man, one of my favorite past-times was hunting the hare. I have enjoyed many Saturdays climbing the hills and knolls of Virginia while on the heels of my beagle. The joy of the hunt was in the chase. I noted that regardless of the path generated by the hare, my well-trained beagle was able to use the curve to converge on the rabbit. He was able to accomplish this amazing feat by using his keen sense of smell and oscillating across the rabbit’s path. Each time the dog intersected the rabbit's path he would decrease the distance between himself and the hare until he was convergent with the hare, as illustrated in Figure 5.
This provides an analogy for conceptual convergence. Think of the techniques of identifying a conceptual gap and then asking questions that zoom ever closer and closer to the concept (intelligence-seeking conversation) as a means of converging the student's perception with a mathematical concept. You are narrowing the learning gap by oscillating back and forth around the target concept with questions and answers. With your keen knowledge of course content and interaction with students your goal is to reach a point of understanding and conceptual convergence.
**Delta-Epsilon Proofs and Convergence**

Conceptual convergence is basically the same logic as used in delta epsilon proofs for finding limits. The formal definition of a limit is as follows:

\[
\lim_{x \to a} f(x) = L \quad \text{iff for each } \varepsilon > 0 \text{ there exists } \delta > 0 \text{ such that if } 0 < |x - a| < \delta \text{ then } |f(x) - L| < \varepsilon.
\]

By choosing smaller and smaller values of delta, I can obtain smaller and smaller values of epsilon until my function converges to my limit.

As an example, I wish to show that

\[
\lim_{x \to 2} (2x + 3) = 7
\]

Consider

\[
|2x + 3 - 7| = |2x - 4| = 2|x - 2| < \varepsilon \quad \text{iff} \quad |x - 2| < \varepsilon/2.
\]

So if I let \( \varepsilon/2 = \delta \), then \(|x - 2| < \delta = \varepsilon/2 \) implies

\[
|(2x + 3) - 7| < \varepsilon
\]

Therefore, \( \lim_{x \to 2} (2x + 3) = 7 \).

In this example by choosing values of delta less than or equal to epsilon divided by 2, I can guide the function to converge to the value of 7.
Conceptual convergence is obtained by decreasing the difference between the student’s perception and the targeted concept. The fundamental steps are:

- Carefully examine the student's written work to identify conceptual errors and learning gaps.
- Use intelligence-seeking conversation to narrow the learning gaps.
- Use extended explanation when necessary.
- Guide student perception to the targeted math concept.

Summary
A number of tutoring techniques can be used to enhance student comprehension. The enlightened tutor will seek to identify learning gaps through a close examination of the student's written work and engaging students in intelligence-seeking conversation. Utilizing open-ended questions will trigger and stimulate student thinking. Through provocative response and listening skills the tutor will be able to link missing concepts with course content. Through extended explanations and intelligence-seeking conversations we can narrow the learning gaps and guide students to converge at conceptual understandings.
Topics for Discussion

- Name 2 elements in Gagne’s theory of instruction.
- What are learning gaps?
- Name two techniques used to construct a bridge for learning gaps.
- Name two techniques used in concept identification.
- List three open-ended questions.
- What are ‘listening links?’
- What is meant by the term “extended explanation?”
- Why are computer generated graphs sometimes insufficient?
- Given the following example problem, write a hypothetical "intelligence-seeking conversation."
  "A projectile travels along the line y = -3x -2. At what point on the line is the projectile closest to the target located at (1,4) ?"
- What are “lesson tips” and where are they found?
- Explain what is meant by the term “Conceptual Convergence.”
T³ Treacherous Tutoring Techniques

In Chapter 3, you learned a number of positive techniques designed to enhance your experience as an Emporium tutor. In this chapter we will look at the negatives -- certain techniques and pitfalls that you should avoid. These negatives are drawn from the experiences of those of us who have learned through personal interaction with tutees on the floor.

Learning to become a good tutor is somewhat like learning to drive a car. To become a good driver, you must learn some positive techniques like k-turns and parallel parking. But, you also need to learn about some dangers. For example, a good driver learns to avoid parking tickets by not leaving their car in a "No Parking" zone, how to steer around potholes to avoid tire damage, and how to adjust the rearview mirror to avoid blinding reflections after dark. Similarly, a good tutor must learn to not only apply positive techniques but also to avoid some treacherous negatives.

No Help Zone
Although the primary role of a tutor is to provide guidance and help, a No Help zone does exist. The blanket policy is that no help of any kind is to be given during a graded quiz or exam. When an exam is proctored, then the No Help
zone is clearly defined. However, quizzes can be confusing because a tutor may help a tutee with a practice quiz but not with a graded quiz.

To help you distinguish between a practice quiz and a graded quiz, a quiz code has been devised to signal when help can or cannot be supplied. The quiz code is listed in the top menu bar of each quiz (graded and practice). You will need to look for and decipher it as you assist students on the floor.

For example, a quiz code might look like this:

$$P \quad 1 \quad 4 \quad 2.5.3$$

This code can be interpreted as Practice Quiz 1, Problem 4, Unit 2, Module 5, Lesson 3. You may help the student with their question.

A second example might look like this:

$$Q \quad 1 \quad 4 \quad 2.5.3$$

This code can be interpreted as Graded Quiz 1, Problem 4, Unit 2, Module 5, Lesson 3. (Note that graded quizzes are also highlighted on the computer screen by being placed against a blue background with a vertical line on the left side of the screen.) You may not help the student with their question.

*What About Incorrect Problems?*

Occasionally, a quiz might have an incorrect problem. For example, the same answer might appear twice in a multiple choice question or vital information may have been omitted in a problem description. The correct procedure for dealing with incorrect problems involves two steps:
1) Have the tutee fill in the best choice that he or she can make, based upon information given in the problem.

2) After submission for grading, the tutor takes a screen shot of the problem including all bar codes on the top of the screen and submits it to the appropriate source for corrections. If it is a graded quiz, a carbon copy is sent to the instructor or professor for grade corrections as well.

Be sure to reassure the tutee that after the quiz has been submitted for grading, the tutee may then contest incorrect problems. If there is a mistake, the instructors can make adjustments as they deem necessary. Also, you might remind students that all of their quizzes are stored in a database for retrieval and review.

Timed Out
Most graded quizzes have a time-out feature that kicks the student out of the program if they spend an undue amount of time on any one problem (30 minutes per problem at the current time of this writing.) They can avoid this if they simply move to a different problem by selecting the number of the problem at the bottom of the quiz page. This information is included in most course syllabi but many students invariably fail to read and be aware of it until it happens to them. The programmers have recently installed a pop-up warning that prompts the students to change problems before they are timed out. You can usually verify this violation by closing the quiz and going into the review past quizzes and tests menu. The time log for the quiz will be shown; if it exceeds 30 minutes this is most probably what happened. The only resolution in this situation is for
the student to send an e-mail to the course coordinator or the instructor concerning the error. You **may not** assure them that they will be granted a re-take; that is left up to the instructor.

*Notify Your Supervisor*

In addition to mistakes in certain quizzes, it is also possible to find a mistake in one of the lessons (from the computer assisted instruction modules). Notify the Emporium supervisor about all such mistakes so as to make this person aware of the error. The supervisor keeps a record of previously reported mistakes. Only when a new mistake is reported is an e-mail sent to course coordinators and programmers. This helps to avoid duplication of reporting and minimizes an undue amount of e-mail to course coordinators and programmers who already work under heavy loads.

*Avoid Hidden Dangers*

Most students are relatively honest and will perform their computing tasks as assigned. Unfortunately, a few students will use some con artist tricks in an attempt to steer you away from noticing that they are taking a graded quiz. One trick is to hide the quiz behind another window on the computer screen. You must keep a sharp lookout for this trick. Carefully check the screen for all open documents before proceeding to help the student.

Another trick is that the student will sometimes move the vertical line located on the left side of a graded quiz out of the window on the computer screen.
Still another trick is that a student will leave their computer pod and approach you somewhere on the floor for information about a problem that he or she has copied down on paper. Upon such requests, you should discreetly inquire as to where the student is working and escort them back to the station. Once there, you can then check to see if a graded quiz is open.

Avoid Holy Ground
"Fools rush in where wise men fear to tread." The tutor should never suggest how instructors or professors will adjust or assign grades. Assigning or adjusting grades is ultimately the responsibility of the course coordinator or the instructor. Know your role. You are the tutor and not the instructor. Therefore, it is not your prerogative or responsibility to suggest such information.

The wise tutor does not voice opinions regarding grading procedures, adjustments or programming errors. Never make negative comments to tutees but rather report errors and suggestions to your administrators and coordinators.

Missed Work and Catching Up
Students have been known to skip classes; some legitimately and some not. They may stay up late to party and over-sleep, actually be ill, or leave town for the weekend and arrive back to campus a day or two late. Upon such occasions they may come to the Emporium and want you to make up the lecture or work they missed.

It is not your responsibility to make up missed materials. You do not want to reinforce negative behavior.
If upon discussion with the student or as a result of intelligence-seeking conversation you suspect the student is totally unprepared, you have the right to discreetly suggest that the student study the lesson module, work through the examples, and then ask for help. *Be firm but always be kind.* It is the responsibility of the tutee to prepare as much as possible before asking for your help.

Students often procrastinate and fail to meet proper quiz and test deadlines. They may try to convince you to give them access to a quiz or some sort of an extension beyond a deadline. You need to refer them to the course schedule and quiz information found on the homepage of their respective course. There are way too many schedules for any tutor to keep up with. *Learn to glean valuable information about course requirements, assignment of grades etc., from the homepage of the instructor.* For more specific questions that may arise, have students e-mail their instructor or course coordinator.

*Avoid Doing the Work for the Student*
Because you know what you are doing, it is incredibly tempting to just grab the mouse and keyboard and show a student how to do something. If you take control, however, you are prohibiting the student from gaining the practice and confidence he or she needs in order to increase retention. Thus, you should avoid as much as possible using the tutee’s mouse, typing with their keyboard, performing calculations on their calculator, and writing down solutions for them.
Try to walk the tutee through the problem with intelligence-seeking conversation and let the tutee maintain control of their system.

Of course, I must admit that there are times when it seems absolutely necessary to take things into hand to help the tutee find a solution to a problem. What alternatives can you use then? One on my colleagues makes the following suggestions:

1. AVOID writing for them.
   INSTEAD, try to get them to write the solution by asking questions with obvious answers. Or, ask them to tell you what to write.

2. AVOID taking over the computer mouse unless you are using it to point out something on a graph or to guide them toward a particular part of a problem.
   INSTEAD, tell them to move the mouse in a certain direction or to trace a specific part of a graph.

3. AVOID finishing the final steps of a problem.
   INSTEAD, leave them with something small to finish on their own. Remember that the correct answer is there for them. Tell them to put the red cup back up if they can’t complete the problem.

Students need to develop confidence to solve problems on their own. That is much easier if THEY got the answer by themselves, even if you guided them through 95% of the solution.

_Hazardous Shortcuts_
A shortcut is designed for people who already know and understand the longer path. To offer a shortcut for solving a
problem without explaining the underlying concept is NOT helpful. In such cases, a shortcut encourages rote memory of some procedure rather than a good conceptual understanding.

The following observation was provided by an instructor of Vector Geometry. It was submitted to us by one of her recitation tutors, a graduate student.

In general, I like answering all types of questions at the Emporium but I have noticed that when a Vector Geometry question comes up, many of the tutors will just tell the students a shortcut without explaining why it works. For example, in a question where the student is asked to solve for initial velocity given maximum range, the response given by a tutor was to "take the range multiply it by 9.8 and take the square root" without any further explanation. This is a great shortcut, if after having worked the problem out the student notices it.

An actual problem of this type in a practice quiz is:

The range is maximum for a given firing speed when the angle is 45°. Find the initial firing speed of a gun fired from ground level whose maximum range is 24, 400 meters.

One proper way to solve this problem is as follows:

Integrate the acceleration vector <0,-9.8> to find the velocity vector
\[ <v_0 \cos 45^\circ, -9.8 t + v_0 \sin 45^\circ>. \]

Integrate the velocity vector to obtain the position vector
\[ \langle v_0 \cos 45^\circ \, t, -4.9 \, t^2 + v_0 \sin 45^\circ \, t \rangle \]

Note that the constants of integration happen to be zero in this particular case but beware that such is not always the case.

Set the \( y \) component of the position vector equal to zero and substitute in the constant \((24, 400)\) for \((v_0 \sin 45 \, t)\) into this expression and then solve for \( t \).

Finally, substitute the value of \( t \) into the \( x \) component of the position vector and solve for the initial velocity.

Clearly, there are numerous integration skills to be gained in this exercise as well as application of the concepts of acceleration, velocity and position vectors. If a tutor gives the student a shortcut, the tutor has circumvented these valuable problem solving strategies and shortcut the student’s learning as well.

One online math instructor reminds us:

*Never show them a "trick" even if it is simply a shortcut. Please remember that this is an on-line course. They do not have a lecture class where someone explains the concepts to them. The only way that they can understand the underlying concept behind a formula is to work with it as it is intended.*

*Also, the quiz problems are randomly generated from a wide variety of available functions. If you show them a trick that works for the one problem you see, it might not work for what the student might see in a different version of the same problem.*
Avoid a Policeman Perception

It is the desire and goal of all those who hold the Math Emporium at heart that students perceive this as a friendly place to receive excellent help with their math. It is not your job to enforce the rules and police for violations on the floor. General Emporium rules do exist. For example, no one is allowed to eat food in the Emporium or drink from containers which do not have lids. When confronted with such issues, feel free to consult the Emporium supervisor who is trained and paid to deal with such matters. We do not wish for our tutees to perceive you as the local marshal carrying a badge.

The obvious exception to this rule is when you are proctoring exams in the designated test area. You are then the "private eye" watching for signs or clues of cheating.

Summary
To become an effective Emporium tutor, you must learn to avoid a number of treacherous situations. Be aware of the No Help zone -- look for and decipher quiz codes in the menu bar so as to avoid giving information at the wrong time. Never answer a tutee’s questions unless you have access to his or her computer screen. Never offer suggestions as to how an instructor or professor may modify a quiz or exam grade.

It is always preferable to let the tutee do the work and perform calculations in order to increase retention rates. Avoid hazardous short cuts which often circumvent basic problem solving strategies. Please remember, that you are the friendly helper and not a police officer.
Topics for Discussion

● Decipher the quiz code (Q 2 5 3.5.4) and comment on its value.

● Why should a tutor be at the tutee’s computer station prior to answering questions?

● What is meant by the term ‘No Help Zone?’

● Describe the process for helping a student with an incorrect problem.

● What is meant by the term ‘Holy Ground?’

● Name 3 things to avoid while helping tutees.

● Explain why shortcuts can be hazardous.
The Power of Positive Reinforcement

Have you heard any of the following complaints?
“'I hate math!'”
“'I cannot do math!'”
“'Math is not my strong subject.'”
“'I hate word problems.'”

Defeatist Attitude
No one enjoys being defeated. Failure breeds more failure. If a student continually fails to solve one problem after another, eventually that student will develop a “defeatist attitude” that becomes a barrier to climbing higher in the mathematics domain.

Students need to build upon successful experiences in their mathematical work in order to gain confidence in attempting other rigorous and more challenging problems. A positive attitude can make a big difference in whether or not the student succeeds. You may recall the story of the little engine that was confronted with the improbable task of climbing a steep path up the side of a high mountain. He built up steam by believing in himself. He repeated again and again, “'I think I can, I think I can ....'” Even though he struggled, he made it to the top of the mountain.
The tutor must constantly search for ways to encourage a positive attitude. As a tutor, your words have tremendous impact upon the students that you are helping. You can encourage or discourage the tutee with your conversation. I recall one of our tutors who had great difficulty getting a particular concept across to a student. The student later e-mailed us that the tutor made the following statement, verbatim, to her.

“I think you should bag this one and go for the next one.”

This had to be devastating for the student. Put yourself in this person’s place and imagine how you would feel if the tutor had said this to you.

**Positive Reinforcement**

One way to help students maintain a positive attitude, and thereby build a bridge to success, is to provide positive reinforcement. Positive reinforcement can be offered in two different ways. First, give verbal praise for good ideas and effort observed in the tutee’s work. Secondly, use body language to communicate and convey in a nonverbal manner that you are pleased with his or her progress.

Learning can be frustrating — especially when you are learning by making mistakes. As a tutor, not only must you learn to point out the tutee’s mistakes but you must also notice their successes. By building on the steps a student has thus far done correctly, you can guide the student to a satisfactory solution.

The TASC, mentioned earlier, provides the following examples of positive reinforcement:
Verbal:
- “Good job on ___________!”
- “You are doing much better with ________!”
- “I like the way you did ________!”
- “This looks better than the last time.”
- “You really have been working hard at this. I am proud of your effort.”
- “All right!”

Nonverbal:
- Use facial expressions—smile, look surprised.
- Nod your head.
- High-five or give a thumbs up sign.

The following excerpt is taken from Guidelines for the Tutor of Mathematics, published by the National Council of Teachers of Mathematics (2001):

Without encouragement many of us would soon give up and may believe that we can’t do it—whatever the “it” is. Your demonstration of encouragement is often necessary…At the focused daily level, a need exists to show the learner that he is making progress at getting something done effectively…Don’t forget to point out, “Look at what you can do now that you couldn’t do last week.”
Summary
Remember, there is power in praise. By praising the student’s accomplishments that you observe, you provide incentive for the student to try again. Find ways to reinforce the student’s progress both verbally and nonverbally. By doing so, you are implementing the tool of positive reinforcement and building a bridge from frustration and confusion to understanding and confidence.

Topics for Discussion

● What is meant by the term “defeatist attitude?”

● Describe two examples of verbal positive reinforcement.

● Describe two examples of nonverbal positive reinforcement.
As an Emporium tutor, you will sometimes be asked to serve as a test proctor. We consider this to be a critical duty. A major purpose of the Math Emporium system is to foster mastery and conceptual understanding of freshmen level mathematics, thus helping students build a foundation for the pursuit of higher learning. The success of the student and the success of the Emporium system are measured in an evaluation process that includes graded quizzes and exams. The students' exam scores not only tell us if they are learning through our program but also reflect whether the tutors at the Emporium are being effective as well. In other words, successful student evaluation leads to successful evaluation of the Math Emporium and its tutors.

You can now understand why it is deemed so important that you do a good job in the testing area. All of our teaching and tutoring culminates at the testing pods. This chapter will describe some of the situations that will likely confront you as a test proctor and offer some solutions to resolve them. In addition, the chapter presents some important information and issues for instructors and professors who use this mode of testing. Your goal in reading this chapter is to become familiar with the proper procedures you need to follow to be effective as a test proctor.
**A Friendly Welcome**

All who work in an academic setting can relate to the pressures we feel from taking tests and exams. It is extremely important that you greet the students who are coming in to take their tests and exams with a friendly smile and a few kind words. You could very well be the pressure-relief valve that relaxes them and helps them to perform their very best. On the other hand if you are cold and indifferent with them you may hinder their performance.

**ID Checks**

There are two times that you must check the student’s identification when proctoring tests. These are:

1) At the beginning of every test and exam.
2) At the end of every test and exam.

The student must present you with a photographic identification card before you can initiate any proctored test. It can be their Student Identification card, a Driver’s license etc. It is your responsibility as a proctor to make sure that the student’s name logged on the computer screen, identification number, and face all agree with the photo id before you administer a proctored test. *This photo id should be left on the pod where it is visible during the entire testing session.* When collecting the test or exam, you should do a follow-up check. Once again, make sure there were no changes in any of these items during the exam period.

If a student fails to produce a photo identification simply explain the policy and ask that he or she return with the
proper identification. If they argue or resist, call for your Emporium Supervisor. **Remember, no ID means no test!**

**Personal Items and Calculators**

Upon arriving at the computer pod the student should only have one piece of paper and writing tools. There are lockers available for students to secure all of their book bags, walkmans, hand-held calculators, cell phones, and other personal belongings at the front of the Math Emporium. The only sheet of paper the student should have is the pod assignment sheet issued at the front desk which tells the student where to sit. Check to see if the student is indeed seated in the proper place.

As a general rule, **no student is allowed to use a hand-held calculator for any proctored test or exam.** There is a calculator in the tool bar located on the desktop of every computer. Students are allowed to use this calculator for proctored tests. *This underscores the importance of encouraging students to practice with this calculator during lesson modules and practice quizzes.* Instructors and professors can help prevent this from becoming problematic by emphasizing this requirement in their class syllabi.

There is one exception to this rule. Students with learning disabilities can apply for and be granted special privileges by the instructor or professor. Upon arrival at the test pod, the student must present the proper form to you, stating they have been granted this privilege. This form must be authorized by the instructor’s signature. If you are in doubt, call for the Emporium Supervisor.
Scrap Paper
When the tutee arrives at the testing pod he or she should have an assignment sheet that was issued from the front desk designating where they should sit. Always check the student ID on this sheet against their photo ID. Please inform the student that this same sheet is to be used as their scrap paper. Many students will use only this sheet but others will want more. If a student needs and requests more paper, you may give them an extra sheet from the official stack of available scrap paper with an Emporium printed label. Students may not use any paper of their own nor should they have any such paper in their possession. Always watch for such extraneous sheets of paper as they may be cheat sheets. You should confiscate any such sheets and save them for documentation if need be.

Honor Code
After the student has logged onto the computer then insert the Test Enabler Disk or USB Key. This will now allow you to launch the proctoring application (Proctored App.) The Honor Code will now appear on the computer screen. Always let the student click the OK button after they have read the Honor Code requirements. By clicking this button they are agreeing to abide by the Honor Code—which is very important.

See Figure 6 to review what is contained in the Honor Code.
In the Honor Code agreement the student agrees not to use the following:

- Instant messaging or chat services on the internet
- Mobile phones
- Formula sheets, books, or any notes
- Hand-held calculators
- Excel, Mathematica, Matlab or other software
- Headphones
- E-Mail

If a student refuses to accept the Honor Code the program will not allow entrance into the test or exam.

*Finished*

When the student has finished, remember to check the photo ID one more time. Collect the scrap paper and place it in the holding box unless the student chooses to keep it. Make sure the student logs off the computer and remind
them to check out at the front desk. This is a good time to look under the keyboard and on the floor for cheat notes.

*Active and Alert*

**You cannot proctor and sit down.** Good proctoring demands that you be on your feet, circulating with a watchful eye at all times. Please do not intimidate students by leaning or peering constantly over their shoulders but you do want them to sense your presence. By being active and alert you can control noise levels at nearby pods which tend to be especially worse on busy nights and help maintain an atmosphere conducive to quiet concentration.

The proctored test area is restricted to students who are working on a test or exam. During an exam, other students may not sit down in the testing area and work on practice quizzes. All practice quizzes and tests are to be completed in the general working area. Once a student sits down in the proctored test area he or she must immediately begin a proctored test or exam. Be active and alert to this kind of violation and send students who want to work on practice material back to the general working area.

*Cheating*

Should I report someone if I catch them cheating on a proctored test? Absolutely! We wish to minimize or eliminate all cheating in the Math Emporium and the only way to do so is to deal with it. If you catch a student cheating on a proctored test, please use the following procedures:

- **Do nothing to call public attention to the student.** If you can quietly confiscate unauthorized papers or
tell the student to shut down unauthorized software, do so.

- Make no accusations, say simply that these are not permitted during testing.
- Allow the student to complete the test undisturbed (aside from the above).
- Record all relevant information, including name, student ID number, course, pod assignment, date and time, and the test or quiz they are taking.
- Document everything that you observe or that transpires, including your interaction with the student and the names of any witnesses.
- If you can discreetly ask another Emporium worker to observe the behavior you have noticed, do so. Do not, however, ask other students in the testing area about the incident.
- If one student points out to you another student who is cheating, thank him or her and write down the student's name and ID number so that student can later be contacted for a witness statement.

As soon as possible after the suspected student has left the testing area, report the incident to the supervisor on duty. Honor code violations should be reported to the Honor System as soon as possible and must be filed within 30 class days of the suspected offense.

*Exam Log- In Failure*

Student anxieties are compounded when they cannot log into the exam. One of the most frequent problems encountered by students is that the computer will not let them log in to the system. This often occurs because
someone has previously disconnected the Ethernet connection or it is loose. If the log-in screen immediately shakes this is one probable cause. There are 3 things you should do:

1) Check to see if the cable is unplugged or loose.
2) If so, plug the connection back into the computer by holding the connector until it gently snaps.
3) Restart the computer.

Another frequent log-in failure occurs when the previous user is still logged onto the computer. You should try re-starting the computer, but in most cases the student must return to the front desk and request another computer.

These troubleshooting techniques can be valuable for general floor use as well.

Exam Errors
Countless thousands of tests and exams are generated in the Math Emporium. Inevitably, an error will occur. Recall that in this situation, the proper procedure is to let the student chose an answer that best satisfies the problem and remind them that after submission they can contest the question, the answer or both. After submission they may take a screen shot and send it to the professor or the instructor through e-mail.

As mentioned earlier, the proctor should also send a screen shot to the respective course coordinator for corrections and carbon copy it to the supervisor who keeps a record of all such errors. If in doubt as to whether this particular problem has already been reported, please check with the supervisor.
Exam Interruption

There is nothing worse or more frustrating for students than to be in the middle of a test or exam and have the software or computer unexpectedly freeze or kick them out of the system. This is a fact of life in a high tech world, so be prepared. Computers are powerful and sophisticated machines but they are susceptible to errors, power failures, user login violations, and browser misuse.

Of course the student wants to recover their exam without loss of valuable work. This is sometimes feasible and sometimes not. To assist the student in attempting a recovery find the button labeled history, located in the working menu on the testing program. Pull down the history menu to where a test or exam is listed with several additional numbers and click. This action will sometimes recover the exam. If you have difficulty with this, feel free to call for the Emporium supervisor.

If the machine has a power failure, it is often because a student has accidentally pulled loose a power cord or something of that nature. The programmers have written a program that will recover tests and exams that are lost due to this problem. Contact the supervisor or computer programmer to execute this recovery program.

If a student is kicked out of the system, this is likely because he or she is still logged in to a practice quiz or test. It matters not where the student is logged onto the system — in the dorm or in the Emporium. The fact remains that if a student fails to log out and proceeds to the testing area to open a test or exam, the system will
automatically kick them out of the test. Your only resolution to this problem is to have the student e-mail the instructor or course coordinator and explain what happened.

If the message “Assessment Error” pops up while a student is engaged in taking a quiz or test, it is likely that the student has used the web-browser's forward or back buttons to navigate through the exam or test or re-sized the window. (Students are warned not to use the forward or back buttons for exams or tests or to resize windows.) By simply clicking “continue” the program should let the student resume the test. If this does not work, call for the supervisor.

Many of these inconveniences can be minimized by professors and instructors informing and admonishing their students of the consequences of these actions in their class orientations and syllabi.

**Tickets**

In case a student experiences an interruption in the exam process and recovery does not work, you may inform the student that they can request a new ticket. A ticket is a new opportunity to initiate a test or exam. These can only be obtained from course coordinators or computer programmers who can add them back into the program. **Never imply to the student that they will automatically be granted a ticket.** This is (once again) holy ground.


**Sharing Passwords**

Students are warned to **never share or give anyone else their password**. This violates every security code on the university campus and has severe consequences.

If a student initiates a proctored test or exam and someone else that has their password goes on-line, the program will once again kick them out of the test or exam. *The test cannot be recovered.* If any of these problems arise you may remind students of these warnings but if they become argumentative please call for your Emporium Supervisor.

**Summary**

As an Emporium tutor, it is your responsibility to learn and follow proper testing procedures. Always check student photo IDs at the beginning and the end of every test. Report all cheating incidents and follow the procedures prescribed. Be aware of why software and computers unexpectedly freeze or kick the student out and how to recover them. Most importantly, be active and alert at all times – stay on your feet and circulate throughout the exam area.
Topics for Discussion

● Why are proctored exams so important?

● When do you check a student’s photo ID?

● Explain the policy on hand-held calculators.

● Why should a test proctor never sit down?

● When do you report a cheating violation?

● What is a “ticket?”

● What are the most probable causes for log-in failure and how do you fix them?

● Explain the procedure to recover an exam.
A Math Emporium system is a complex infrastructure that integrates tutoring models, learning and instructional theories, and modern technology. The Emporium tutoring model is bold, thrives, and sets into motion one of the most advanced approaches to math education in the 21st century.

Different students learn in different ways and respond best to different instructional approaches. Some learn best by auditory instruction while others respond better to visual enhancement. Some learn better with hands-on experience, others with group instruction, and still others with peer interaction or individualized instruction. The Emporium gathers all these strategies and streams them into one dynamic resource for teaching and learning. It gathers the best from all of these instructional models and distributes them through a diversified and qualified staff of tutors.

As an Emporium tutor, you are expected to learn and apply the techniques, guidelines, and procedures presented in this manual. Be consistently prepared, positive and enthusiastic. You can help to bridge the learning gaps for students from confusion and misconception to understanding and conceptual convergence. The techniques you have been offered include: positive reinforcement, identification of learning gaps, intelligence-seeking conversation, listening links, and extended explanation.
The capacity and potential for success in this system is as effervescent as the fountain of mathematics from which it continually draws. Your role as a tutor is crucial. The purpose of this manual is to increase your power (help you be more effective) as a tutor.

No doubt, you have noticed that in the scheme of this booklet the degree or powers of the topics have lagged one degree behind the number of the chapters. This has symbolic significance in the deliberate design of this composition. In this final summary, please allow me to elaborate on this design.

- This design is significant with regard to learning and instructional theory, which will perhaps always lag one step behind the rapid advance of technology. We must continually strive to catch up. Never become complacent and satisfied with where you are in the scheme of math education.
- The powers of math educators and tutors are ever increasing.
- You alone, through your self determination and desire to excel, can supply the missing power to transform the Emporium experience into a successful one.
ADDENDUM

The following is a list of (Macintosh) computer commands that you will find helpful while assisting students in the Math Emporium.

**Brighten Screen**- Hold down the F15 key

**Calculator Retrieval**- If the calculator does not appear in the dock, go to the Applications menu and select “Slide Rule” for the calculator.

**Force Quit** - Click the blue apple in the upper left hand corner of the screen (if it is available) and then select the force quit button.
If the blue apple button is unavailable, click
  Apple + Option + Escape

**Grab**- This action requires several steps:
  Go to the dock and select “Applications.”
  Double click “Grab”
  From the top menu bar click "Capture."
  From the pull down menu click “Selection.”
  Position the mouse on the upper left corner of the picture and drag a red box over the portion of the graph you want to copy, then release. (The picture will pop up on the screen.)
  Go to edit and use copy and paste.
**Image Restoration**- (Useful when you encounter a student with images in a quiz or other web page that do not load or load in a scrambled manner):
Hold down the “Control” key and click the image once.
From the Popup menu select “Load Image.”

*Note: If this fails you may have to reload the quiz or restart the computer.*

**Log Out**- Click- (Shift+ Apple + ‘Q”) or select “Log Out” from the desktop.

**Printer Restoration**- This action is needed when the computer cannot seem to find the printer. It requires several steps:

Click on the desktop.
In the “Go” menu select “Applications.”
Scroll down to “Utilities” folder and double click.
Scroll down to “Print Center” and double click.
Select “Add Printer.”
Select “IP Printing” from the pull down menu.
In the Printer’s Address field, enter the code 128.173.42.228
Uncheck the box “Use Default Queue on Server” in the Queue Name field enter “EMP1” (no space between the P and the 1).
Select “HP” from the Printer model pull-down menu.
Scroll down and select “HP Laserjet 5 Si/5Si MX Postscript” as the model name.
Click “Add”
Quit the Print Center.
**Retrieval of Lost Documents**- (Useful when a student logs out but then wants to retrieve a document.) Go to “Finder” in the dock and select “Old Files.” Then select the document to be retrieved.

*Note*- This only works for one log out.

**Screen Shot**- This action creates a picture on the desktop and requires two steps:
Click- (Shift + Apple + ‘4’). A “+” sign will appear on the screen. 
Move the mouse to the upper left hand corner of the document to be captured and drag over the desired portion and release.

*Note*- When copying quiz or test errors make sure you include all codes at the top of the screen.
References


